Introduction
Since exponential stretching is typical for laminar mixing of fluids, a special numerical technique for handling both macro- and micro-structure evolution is desirable for simulation purposes.

Extended mapping technique
The flow domain is subdivided into a small cells and the mixture described by concentrations in these cells. Transport of material between cells during a (large) time step is given by a pre-computed sparse mapping matrix (the method applies to rheologically identical fluids with negligible surface tension). Each cell receives from donor cells both concentration and interfaces (Fig. 1).

Fig. 1 Mapping of concentration and microstructure
The microstructure is described by the area tensor $A$:

$$ A_{ij} = \frac{1}{V_{cell}} \int n_i n_j dS, $$

where $n$ is the a unit normal to the increment of interfacial area $dS$.

Fig. 2 Definition of area tensor
The interfacial area per unit volume is given by $\text{tr}A = s_v$. Examples of different types of mixture and corresponding area tensors are shown in Fig. 3.

Fig. 3 Area tensor for different mixture morphology
The area tensor is mapped and properly transformed under finite deformation [1].

Example: cavity flow
Periodic Stokes flow ($\text{Re} \ll 1$) in a rectangular cavity with alternately sliding horizontal walls (top to the right, bottom to the left by 8 times their length), was examined. This flow is globally chaotic [2].

Fig. 4 Evolution of concentration and $\log(\text{tr}A)$. Zeros of $\text{tr}A$ are replaced by small values to enable log. plots.

The macroscopic variations in concentration quickly disappear, while on microscopic level the mixture remains structured. Distribution of interfaces becomes self-similar and interfacial area grows exponentially.

Conclusions
Extended mapping provides direct multiscale simulations, handling both macroscopic transport and evolution of microstructure in laminar mixing. This method treats both initial and advanced stages of mixing. This makes it useful engineering tool.

References: